Modeling Diffusion Between Regions With Different Diffusion Coefficients

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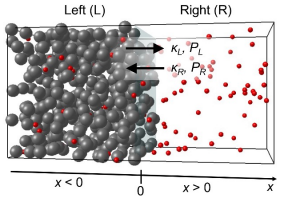
Main Contribution: To accurately model diffusion between regions with different diffusion coefficients, such as into phase-separated cytoplasmic droplets, the simulator needs to modify the transmission probabilities.

The problem

Diffusion coefficients vary over space in many cellular and engineered systems, as in cytoplasmic droplets and lipid rafts.



In this figure, diffusion is slower on the left, due to crowding, and faster on the right.

How does this variation affect particle concentrations?

- Intuition might say that particles accrue on the slow side.
- But, if particles have the same energy on both sides, as they do in most cellular contexts, then physics says that the average concentration *must* be the same on both sides. Simulations with explicit crowders agree with this.

The problem. If diffusion changes are treated implicitly (by using different diffusion coefficients) and if simulators allow particles to cross the boundary with 100% probability in each direction, then simulated particles accrue on the slow side. This is not physically correct.

Solution

At equilibrium, net flux across the interface is zero, so the transmission coefficients, κ_L and κ_R, relate to the concentrations, C_L° and C_R°, as

$$\kappa_R C_R^\circ = \kappa_L C_L^\circ$$

In a simulation of the same system over time Δt, fluxes are

$$\text{amount from left} = P_L C_L^\circ \sqrt{\frac{D_L \Delta t}{\pi}}$$

$$\text{amount from right} = P_R C_R^\circ \sqrt{\frac{D_R \Delta t}{\pi}}.$$

D_L and D_R are diffusion coefficients and P_L and P_R are transmission probabilities. Equating these and combining with above gives the correct simulator transmission probabilities,

$$\frac{P_L}{P_R} = \frac{\kappa_L}{\kappa_R}\sqrt{\frac{D_R}{D_L}}.$$

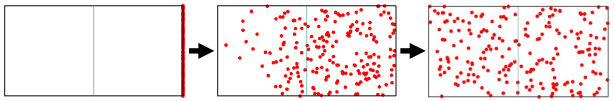
- If no membrane: $\kappa_L = \kappa_R$ and $P_L = 1$; use these to compute P_R.
- With a membrane:

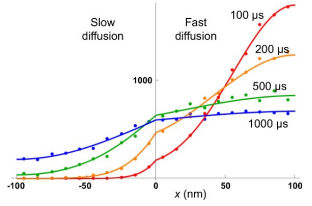
$$P_L = \frac{\kappa_L \sqrt{\pi \Delta t}}{c^2 \sqrt{D_L}}\left(-1 + \frac{2c}{\sqrt{\pi}} + e^{c^2}\operatorname{erfc} c\right) \qquad c = \sqrt{\Delta t}\left(\frac{\kappa_R}{\sqrt{D_R}} + \frac{\kappa_L}{\sqrt{D_L}}\right)$$

and then compute P_R.

Validation

A system has slow diffusion on the left and fast diffusion on the right. Molecules start at the far right edge and diffuse.





Concentrations over time:

- Dots: simulations
- Lines: exact theory

Agreement is excellent.

Methods. All simulations were run in Smoldyn 2.72. Smoldyn now supports these algorithms automatically.

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Reference. Andrews, Steven S. "Modeling diffusion between regions with different diffusion coefficients" IEEE Transactions on Molecular, Biological, and Multi-scale Communications, 10:425-432, 2024.