World of Light - Problem Set #2

Assigned April 8, due at start of class on ***Wed. April 15***.

Reading

Light Science chapters 1 and 2.

Topics and Equations

This problem set reviews the following topics: resonance and coupling, spectra, absorption, transmission, unit analysis, unit conversion, and shadows.

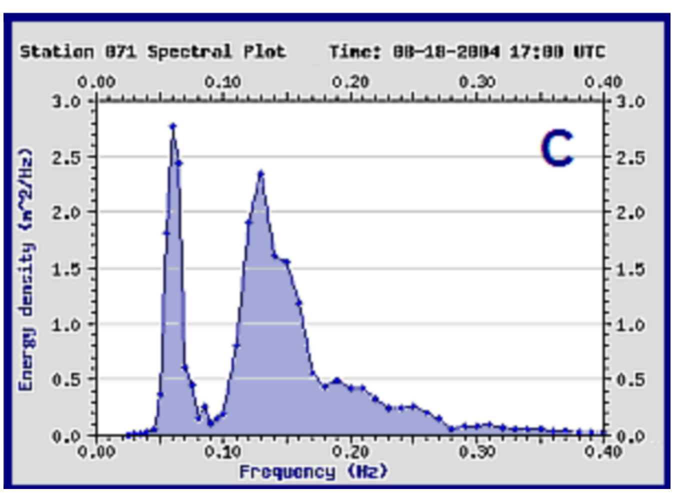
Problems

Grading scale: basically right = 1 point, basically wrong = 0 points, some right and some wrong = 0.5 points.

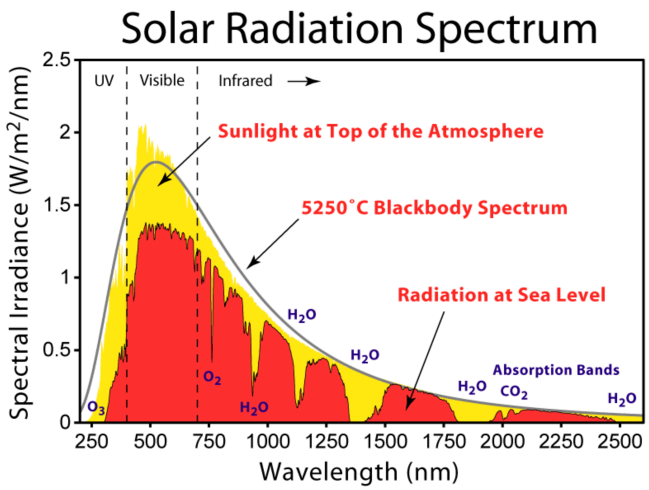
1. Consider a bathroom shower that is 1.5 m long and 1.5 m wide, and suppose you're singing in this shower. (a) What sound wavelength has the lowest resonant frequency? That is, its nodes are at the shower walls and its antinode is in the shower center. (b) Using the fact that the speed of sound in air is 350 m/s, what is the frequency of this sound wave? (c) To the nearest integer, how many octaves is it above or below A440 (The A node which is above middle C and has a frequency of 440 Hz)? An octave is a factor of 2 in frequency.

2. It is possible for a sustained high musical note to break a wine glass through resonance (check this out on youtube if you want). (a) What two periodic processes are in resonance with each other? (b) Is this favored by strong or weak coupling? (c) Is this favored by strong or weak damping?

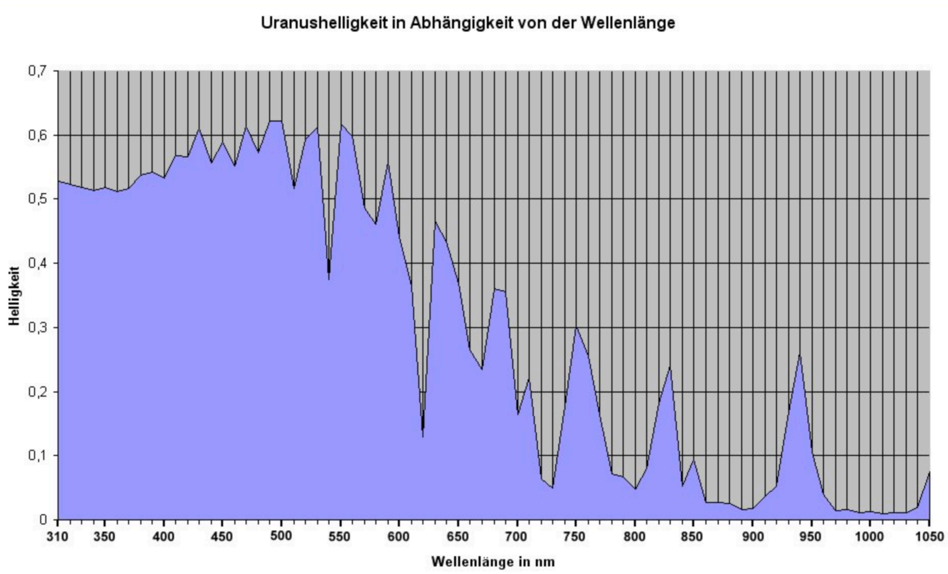
3. Shown below is a water wave spectrum, measured by the Coastal Data Information Program at UCSD. It shows the energy at different wave frequencies, measured by an oceanographic buoy. (a) What are the peak frequencies of the two swells? (b) What are these two wave periods? (c) Which swell has a higher peak energy? (d) Which swell has a higher total energy? (e) *Extra credit*: What are the wavelengths and velocities for the two swells?



4. Shown below is the solar radiation spectrum. The yellow curve is the sunlight measured in space, above the atmosphere, and the red curve is the sunlight measured at sea level. Ignore the black curve. (a) At what color is the sun brightest both above the atmosphere and at sea level (hint: compare the peaks to 500 nm)? (b) Sunlight is usually described as white; can you explain why? (c) Estimate the atmospheric transmission coefficient (in percent) at 300 nm, 500 nm, 1400 nm, and 1600 nm.



5. Shown below is the Uranus brightness spectrum. This is the color of Uranus, as seen by a telescope in space above our atmosphere. (a) Using this, what color is Uranus? (b) By comparing this spectrum with the solar irradiation spectrum from above, is Uranus absorbing the most sunlight in the red, green, or blue?



6. Carotene is a chemical from carrots and pumpkins and is a major component of vitamin A. Electrons in a single molecule of carotene have a natural oscillation frequency of 6.7×10^{14} Hz. Carotene absorbs light with this frequency. (a) What

wavelength of light does this correspond to? (b) What color does carotene absorb? (c) What color does carotene reflect?

7. Unit conversion practice. For each problem, use the method presented here and show your work. Example problem: how many feet are in a meter? Answer:

$$? \text{ feet} = \frac{1 \text{ m}}{1} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ inch}}{2.54 \text{ cm}} \cdot \frac{1 \text{ foot}}{12 \text{ inch}} = 3.28 \text{ feet}$$

(a) How many km is 4000 miles? (b) How many nm is the width of a human hair? (c) How many cycles per day is 97 MHz? (d) How many gallons are in 0.25 acre-feet (the annual water use of a typical family)? (e) How many km/liter are equal to 40 miles per gallon? (f) How many US$/gallon is equal to 1.28 euros/liter (the price of gas in France)?

Unit conversions:

1 inch = 2.54 cm	1 minute = 60 seconds
1 foot = 12 inches	1 hour = 60 minutes
1 mile = 5280 feet	1 day = 24 hours
1 human hair width = 17 μm	1 gallon = 3.785 liters
1 acre = 43560 ft^2	1 euro = 1.14 US$
1 ft^3 = 7.48 gallons	

8. You are casting a shadow of a ball onto a wall. (a) How can you make the umbra larger than the ball (or is it impossible)? (b) How can you make the umbra smaller than the ball (or is it impossible)? (c) How can you make the radius of the penumbra larger than the ball's radius (or is it impossible)? (d) How can you make the radius of the penumbra smaller than the ball radius (or is it impossible)?

9. Following are several equations. For each, simply calculate what the units are for the result, if possible; if it's not possible, write "invalid equation". Don't worry about the numerical values.
(a) $E = hf$ (photon energy). h is in J·s, f is in Hz.
(b) $P = 1/d_o + 1/d_i$ (curved mirror equation). d_o and d_i are in m.
(c) $\lambda = g/(2\pi f^2)$ (deep water wavelength). g is in m/s^2, f is in Hz.
(d) $E = \sigma T^4$ (Stefan-Boltzmann law). σ is in W·m^{-2}K^{-4}, T is in K (note that 1 W = 1 J/s).
(e) $\tau = L/[v(1-R)]$ (laser cavity decay time). L and R are in m, v is in m/s.
(f) $h_r = h_a/\lambda$ (relative water depth). h_a and λ are in m.

10. Consider an acoustic guitar with 65 cm long strings (which is typical). The A string is tuned to produce a note with a frequency of 110 Hz. (a) How many octaves is this below "middle A", or A440. (b) What is the wavelength of the lowest resonant normal mode? (c) What is the wave velocity on the string? (d) What is the frequency of the first overtone? (e) To get a frequency of 123 Hz (a 'B' note), how long should the string be?