Homework 8

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9.1 Consider a sytem consisting of two particles, each of which can be in any one of three quantum states of respective energies 0, ϵ, and 3ϵ. The system is in contact with a heat reservoir at temperature T.
(a) Write an expression for the partition function Z if the particles obey classical MB statistics and are considered distinguishable.
(b) What is Z if the particles obey BE statistics?
(c) What is Z if the particles obey FD statistics?

9.2 (Edited from Reif.)
(a) From a knowledge of the partition function Z derived in the text, along with the entropy for a canonical ensemble and the Fermi-Dirac distribution, show that the entropy S of an ideal FD gas is $S = -k \sum_r [(1 - \bar{n}_r) \ln(1 - \bar{n}_r) + \bar{n}_r \ln \bar{n}_r]$.
(b) Derive a similar expression for the entropy S of a BE gas.
(c) What do these expressions for S become in the classical limit when $\bar{n_r} << 1$?

9.12 (Edited from Reif.) It has been reported that a nuclear fission explosion produces a temperature of the order of 10^6 K. Assuming this to be true over a sphere 0.1 m in diameter, calcuate:
(a) the total rate of electromagnetic radiation from the surface of this sphere;
(b) the radiation flux (power incident per unit area) at a distance of 1 km;
(c) the wavelength corresponding to the maximum in the radiated power spectrum. What type of radiation is this (e.g. infrared, visible, ultraviolet, X-ray, etc.)?

9.13 (Edited from Reif.) The surface temperature of the sun is $T_S = 5778$ K, its radius is $R = 7 \cdot 10^8$ m, the radius of the Earth is $r = 6.37 \cdot 10^6$ m, the mean distance between the sun and the Earth is $L = 1.5 \cdot 10^{11}$ m, and Earth's albedo is $\alpha = 0.3$, meaning that it reflects 30% of incident light and absorbs the other 70%. The temperature of the cosmic background radiation is actually $T_c = 2.7$ K but you may assume it is 0 K for this problem. The Earth has reached a steady state so that its mean temperature T does not change in time despite the fact that the Earth constantly absorbs and emits radiation.
(a) Find an approximate expression for the temperature T of the earth in terms of the astronomical parameters mentioned above.
(b) Calculate this temperature T numerically and give the answer in both Kelvin and Celsius.
(c) List the main factor that would need to be included to make this calculation more accurate.

9.16 (Edited from Reif.) An ideal Fermi gas is at rest at absolute zero and has a Fermi energy μ. The mass of each particle is m. If $\boldsymbol{v}$ denotes the velocity of a molecule that has energy equal to the Fermi energy, find (a) $\langle v_x \rangle$ and (b) $\langle v_x^2 \rangle$. (c) Compute the root mean square velocity, $\sqrt{\langle \boldsymbol{v} \rangle}$ for copper, which has a Fermi energy of 7.00 eV $= 1.12 \cdot 10^{-18}$ J. The mass of an electron is $9.11 \cdot 10^{-31}$ kg.

9.17 Consider an ideal gas of N electrons in a volume V at absolute zero.
(a) Calculate the total mean energy $\bar{E}$ of this gas. (Hint: use Reif eq. 9.9.19, and the fact that electrons have 2 spin states.)
(b) Express $\bar{E}$ in terms of the Fermi energy μ.

(c) Show that $\bar{E}$ is properly an extensive quantity, but that for a fixed volume V, $\bar{E}$ is not proportional to the number N of particles in the container. How do you account for this last result despite the fact that there is no interaction potential between the particles?