Homework 5

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5.1 An ideal gas has a temperature-independent molar specific heat c_V at constant volume. Let $\gamma = \frac{c_p}{c_V}$ denote the ratio of its specific heats. The gas is thermally insulated and is allowed to expand quasi-statically from an intial volume V_i at temperature T_i to a final volume V_f.
(a) Use the relation $pV^\gamma =$ constant to find the final temperature T_f of this gas.
(b) Use the fact that the entropy remains constant in this process to find the final temperature T_f. (Hint for part b: see the solution to problem 4.3.)

5.3 An ideal diatomic gas has a molar internal energy equal to $E = \frac{5}{2}RT$ which depends only on its absolute temperature T. A mole of this gas is taken quasi-statically first from state A to state B, and then from state B to state C along the straight line paths shown in the diagram of pressure p versus volume V.
(a) What is the molar heat capacity at constant volume of this gas?
(b) What is the work done by the gas in the process $A \to B \to C$?
(c) What is the heat absorbed by the gas in this process?
(d) What is its change of entropy in this process? (Note that 10^6 dynes/cm^2 = 10^5 Pa and 10^3 cm^3 = 10^{-3} m^3).

5.7 Consider the Earth's atmosphere as an ideal gas of molecular weight μ in a uniform gravitational field. Let g denote the acceleration due to gravity.
(a) If z denotes the height above sea level, show that the change of atmospheric pressure p with height is given by $\frac{dp}{p} = -\frac{\mu g}{RT}dz$ where T is the absolute temperature at the height z.
(b) If the decrease of pressure in (a) is due to an adiabatic expansion, show that $\frac{dp}{p} = \frac{\gamma}{\gamma-1} \cdot \frac{dT}{T}$.
(c) From (a) and (b) calculate $\frac{dT}{dz}$ in degrees per kilometer. Assume the atmosphere to consist mostly of nitrogen (N_2) gas for which $\gamma = 1.4$.
(d) In an isothermal atmosphere at temperature T, express the pressure p at height z in terms of the pressure p_0 at sea level.
(e) (Extra credit.) If the sea-level pressure and temperature are p_0 and T_0, respectively, and the atmosphere is regarded as adiabatic as in part (b), find again the pressure p at height z. (Hints: For (a), think about the pressure at the top and bottom of a small volume of air; for (e), use $\int \frac{dx}{1+Ax} = \frac{\ln(1+Ax)}{A}$)

5.19 The van der Waals equation for 1 mole of gas is given by $(p + av^{-2})(v - b) = RT$. In general, curves of p versus v for various values of T exhibit a maximum and a minimum at the two points where $(\frac{\partial p}{\partial v})_T = 0$ (the curves are similar to those of Fig. 8.6.1). The maximum and minimum coalesce into a single point on the curve where $(\frac{\partial^2 p}{\partial v^2})_T = 0$ in addition to $(\frac{\partial p}{\partial v})_T = 0$. This point is called the "critical point" of the substance and its temperature, pressure, and molar volume are denoted by T_c, p_c, and v_c, respectively.
(a) Express a and b in terms of T_c and v_c.
(b) Express p_c in terms of T_c and v_c.
(c) Write the van der Waals equation in terms of the reduced dimensionless variables $\tilde{T} = \frac{T}{T_c}$, $\tilde{v} = \frac{v}{v_c}$, and $\tilde{p} = \frac{p}{p_c}$.

5.22 Refrigeration cycles have been developed for heating buildings. The procedure is to design a device which absorbs heat from the surrounding earth or air outside the house and then delivers heat at a higher temperature to the interior of the building. (Such a device is called a "heat pump.")
(a) If a device is used in this way, operating between the outside absolute temperature T_o and an interior absolute temperature T_i, what would be the maximum number of kilowatt-hours or heat that could be supplied to the building for every kilowatt-hour of electrical energy needed to operate the device?
(b) Obtain a numerical answer for the case that the outside temperature is 0°C and the interior temperature is 25°C.