

## PHYS-1050-01,02 Final exam review guide

The final exam will be about 50% on chapter 8 and 50% on prior material. This review guide covers only chapter 8; see other review guides for the other material.

### **Chapter 8 - Rotational Motion**

Angular position is measured with angle  $\theta$ . The equations in this chapter assume  $\theta$  is in radians. 1 rotation =  $2\pi$  radians =  $360^\circ$ .

Arc length,  $l$ , is  $l = r\theta$ . This is also a good approximation for straight line length at distance  $r$  if  $\theta$  is small.

Angular velocity is  $\omega = \Delta\theta/\Delta t$ , measured in either rad/s or  $s^{-1}$ .

Period,  $T$  is time for 1 period. Frequency,  $f$ , is rotations per time,  $f = 1/T = \omega/2\pi$ . 1 rpm is 1 rotation per minute and 1 Hz is 1 rotation per second.

Angular acceleration is  $\alpha = \Delta\omega/\Delta t$ , measured in either  $\text{rad/s}^2$  or  $s^{-2}$ .

From arc length, velocity at  $r$  is  $v = r\omega$  and tangential acceleration at  $r$  is  $a_{tan} = r\alpha$ .

Don't forget that centripetal acceleration is still  $a_{cent} = v^2/r$ .

Total acceleration is vector sum:  $\mathbf{a}_{total} = \mathbf{a}_{tan} + \mathbf{a}_{cent}$ .

Rotational kinematics equations are essentially the same as those for linear kinematics.

Torque is the rotational force. It is  $\tau = F_{\perp}r = Fr_{\perp} = Fr \sin\theta$ . Measured in N·m with the interpretation of being  $F$  newtons at a radius of  $r$  meters (in foot-pounds in English system).

From Newton's second law,  $\tau = I\alpha$ . More precisely,  $\sum \tau = I\alpha$ , because it is net torque that matters.

Moment of inertia,  $I$ , depends on mass and mass distribution. For an object of mass  $M$  at radius  $R$ ,  $I = MR^2$ . Units are  $\text{kg}\cdot\text{m}^2$ .

Objects with mass closer to the center have lower moments of inertia. Examples: a disk has  $I = 1/2 MR^2$ , a rod has  $I = 1/12 ML^2$ , a sphere has  $I = 2/5 MR^2$ , etc.

Moment of inertia depends on the rotational axis.

For objects with multiple components,  $I_{total} = I_1 + I_2 + \dots$  (assuming same rotational axis).

Rotational kinetic energy is  $KE_{rot.} = I\omega^2/2$ , much like  $KE_{trans.} = mv^2/2$  for translational case.

Energy is in J, as always.

Total kinetic energy is  $KE_{total} = KE_{trans.} + KE_{rot.}$ .

Rotational work is  $W = \tau\Delta\theta$ , much like  $W = F\Delta x$  for translational case.

Angular momentum is  $L = I\omega$ , much like  $p = mv$  for translational case. Units are  $\text{kg}\cdot\text{m}^2/\text{s}$ .

In the absence of external forces, angular momentum is conserved.

The following quantities are scalars:  $T, f, I, KE, W$ .

The following quantities can be vectors:  $\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\tau}, \mathbf{L}$ . The vector direction is parallel to the rotational axis, found using the right-hand rule.